Comment on "Aging, phase ordering and conformal invariance"

In a recent letter [1] Henkel, Pleimling, Godrèche and Luck (HPGL) have derived the linear response function of aging systems in the scaling regime of a quench at T_C or below T_C obtaining $R(t,s) \sim s^{-1-a} x^{1+a-\lambda/z}(x-1)^{-1-a}$ where z and λ are the dynamic and the Fisher and Huse [2] exponent, a is the *response* exponent and $x = t/s$. While z and λ are well known for most systems, relatively little is known about a. Here we will restrict to the quench below T_C. Scaling of $R(t,s)$ [3] implies scaling of both the zero field cooled magnetization (ZFCM) $\chi(t,t_w) = \int_{t_w}^{t} ds R(t,s) \sim t_w^{-a_\chi} F(t/t_w)$ and of the thermoremanent magnetization (TRM) $\rho(t,t_w) = \int_0^{t_w} ds R(t,s) \sim t_w^{-a_\rho} E(t/t_w)$ with $a_\chi = a_\rho = a$. Assuming that scaling of the TRM holds in the time range of their simulations HPGL have measured a_ρ in the ferromagnetic Ising model obtaining $a_\rho = 1/2$ for $d = 2, 3$ (with logarithmic corrections for $d = 2$). On the other hand, putting together exact results for the $d = 1$ Ising model [4], for the large N model [5] and numerical simulations of the Ising model for $d = 2, 3, 4$ [6] we have found a different result for the ZFCM exponent

$$a_\chi = \begin{cases} \theta(d-d_L)/(d_U-d_L) & \text{for } d < d_U \\ \theta & \text{for } d > d_U \end{cases} \tag{1}$$

with $\theta = 1/2, d_L = 1, d_U = 3$ for Ising, $\theta = 1, d_L = 2, d_U = 4$ for large N and logarithmic corrections at d_U. Therefore, concentrating on the Ising model, there is a large discrepancy for $d = 2$ where HPGL find $a_\rho = 1/2$ with logarithmic corrections as opposed to $a_\chi = 1/4$ and a lesser one for $d = 3$ where $a_\rho = 1/2$ is opposed to $a_\chi = 1/2$ with logarithmic corrections. It is the purpose of this comment to show that the correct value of a is given by a_χ and that the discrepancy with a_ρ is due to a large preasymptotic correction affecting TRM in the range of t_w explored by HPGL. In order to do this we have done over again the simulations of the Ising model in the same conditions of HPGL, measuring both TRM and ZFCM over a wider range of t_w. The scaling collapse of the ZFCM data as t_w is varied allows to identify a_χ. We find scaling with $a_\chi \simeq 0.27$ for $d = 2$ and $a_\chi \simeq 0.48$ with possible logarithmic corrections for $d = 3$ (Fig.1), consistently with Eq. (1).

Next, in order to analyze the TRM data, let us borrow from the large N model [5] the general form $R(t,s) = r_0[Y(s)/Y(t)](t-s)^{-1-a}$ which contains scaling taking $Y(t) \sim t^{-1-a+\lambda/z}$ for times larger than a characteristic microscopic time t_0. Then, for $t \gg t_w$ one has $\rho(t,t_w) = H(t_w)E(t/t_w)$ with $E(x) \sim x^{-\lambda/z}$. About $H(t_w)$ not much can be said if $t_w < t_0$. Instead, if $t_w > t_0$ one has $H(t_w) \sim t_w^{-\lambda/z}[1+(t_w/t^*)^{\lambda/z-a}]$ where t^* is the crossover time from $H(t_w) \sim t_w^{-\lambda/z}$ to $H(t_w) \sim t_w^{-a}$. On the other hand, a_ρ is defined by $H(t_w) \sim t_w^{-a_\rho}$. Therefore, if $t^* > t_0$ there are three regimes: 1) early regime for $t_w < t_0$ where a_ρ is not well defined, 2) preasymptotic regime $t_0 < t_w < t^*$ where $a_\rho = \lambda/z$, and 3) asymptotic regime $t_w > t^*$ where $a_\rho = a$. Our data (Fig.1) show that even with the largest t_w we have reached the asymptotic regime has not been entered, since we find an early regime followed by a power law with an exponent a_ρ which compares well with λ/z ($\lambda/z = 5/8$ for $d = 2$ and $\lambda/z = 3/4$ for $d = 3$). The range of t_w explored by HPGL is in between the early and the preasymptotic regime where the slope of $H(t_w)$ is compatible with an effective exponent close to 1/2 both for $d = 2$ and $d = 3$, but this is in no way related to the asymptotic value of a_ρ.

In conclusion, our results show that the measurement of the response exponent from TRM requires exceedingly large t_w, while it is manageable from ZFCM. We conjecture Eq. (1) to be of general validity for coarsening systems and work is under way to check it with conserved dynamics.

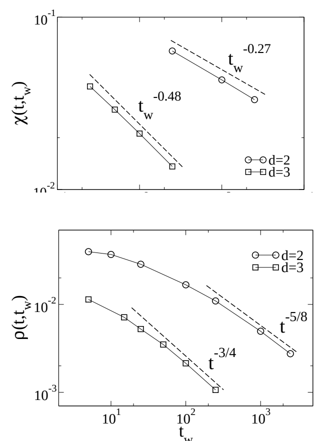

FIG. 1. $\chi(t,t_w)$ (left) and $\rho(t,t_w)$ (right) are plotted against t_w for fixed $t/t_w = 10$. The same behavior is found for different values of t/t_w.

F.Corberi, E.Lippiello and M.Zannetti

Istituto Nazionale per la Fisica della Materia, Unità di Salerno and Dipartimento di Fisica "E.R.Caianiello", Università di Salerno, 84081 Baronissi (Salerno), Italy

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