Disorder-induced rounding of the phase transition in the large-q-state Potts model

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Motivations

- CRITICAL PROPERITES OF DISORDERED SYSTEMS
- EFFECTS OF DISORDER ON FIRST ORDER PHASE TRANSITIONS
- $\Longrightarrow$ RANDOM BOND POTTS MODEL (RBPM), ESPECIALLY IN THE LARGE-q-LIMIT, IS A PERFECT GROUND TO ANALIZE THIS ASPECT

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Outline

- POTTS MODEL IN THE RANDOM CLUSTER REPRESENTATION
- INTRODUCING DISORDER
- RESULTS AND PERSPECTIVES

q-state Potts model

$$Z \equiv \sum_{\{\sigma\}} q^{-\beta\mathcal{H}(\{\sigma\})}$$

$$\mathcal{H} = -\sum_{\langle i,j\rangle} J_{ij}\delta(\sigma_i,\sigma_j) \qquad \sigma_i = 0,1,\cdots,q-1$$

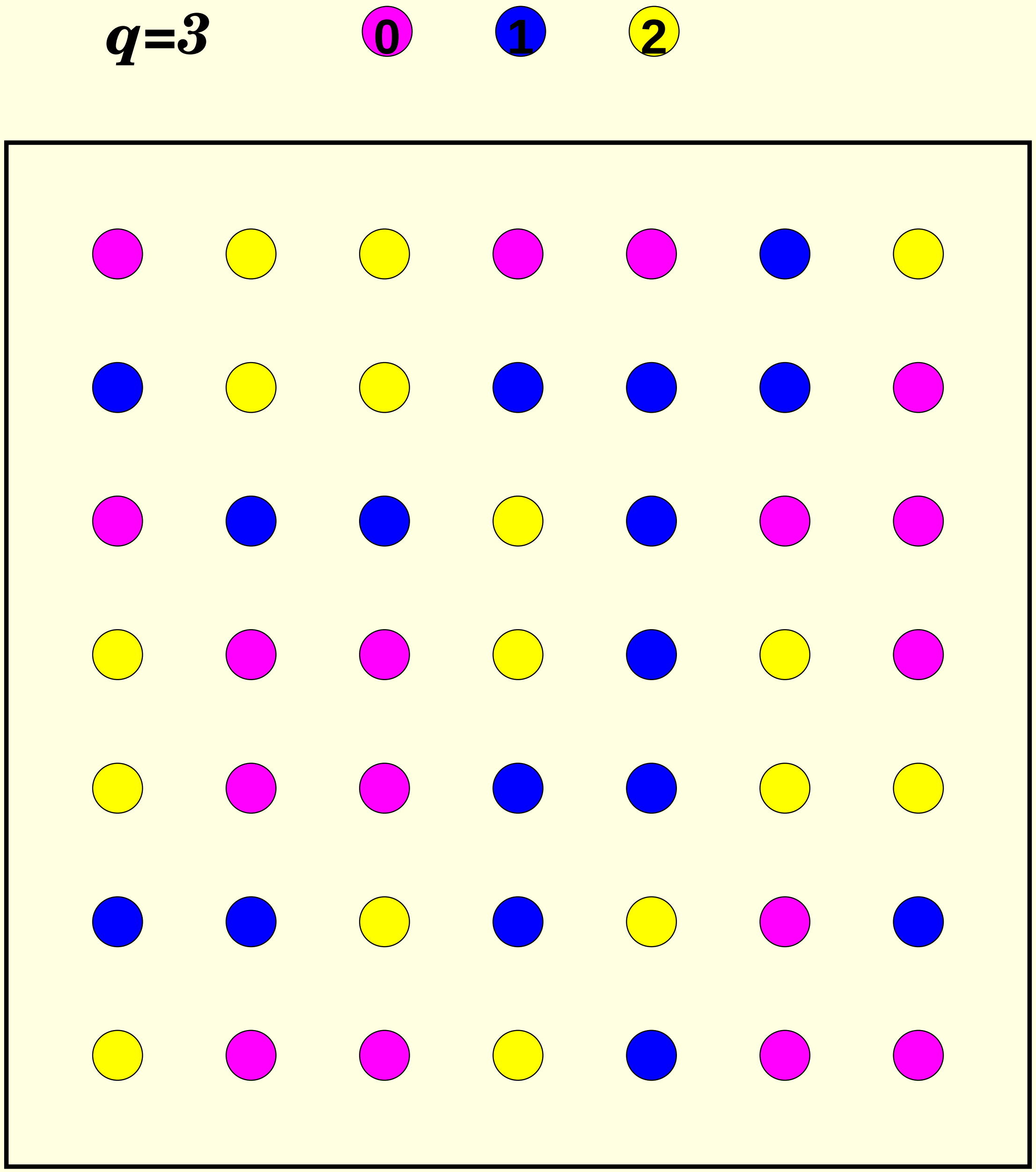
J_{ij} FM random couplings

$$\Downarrow$$

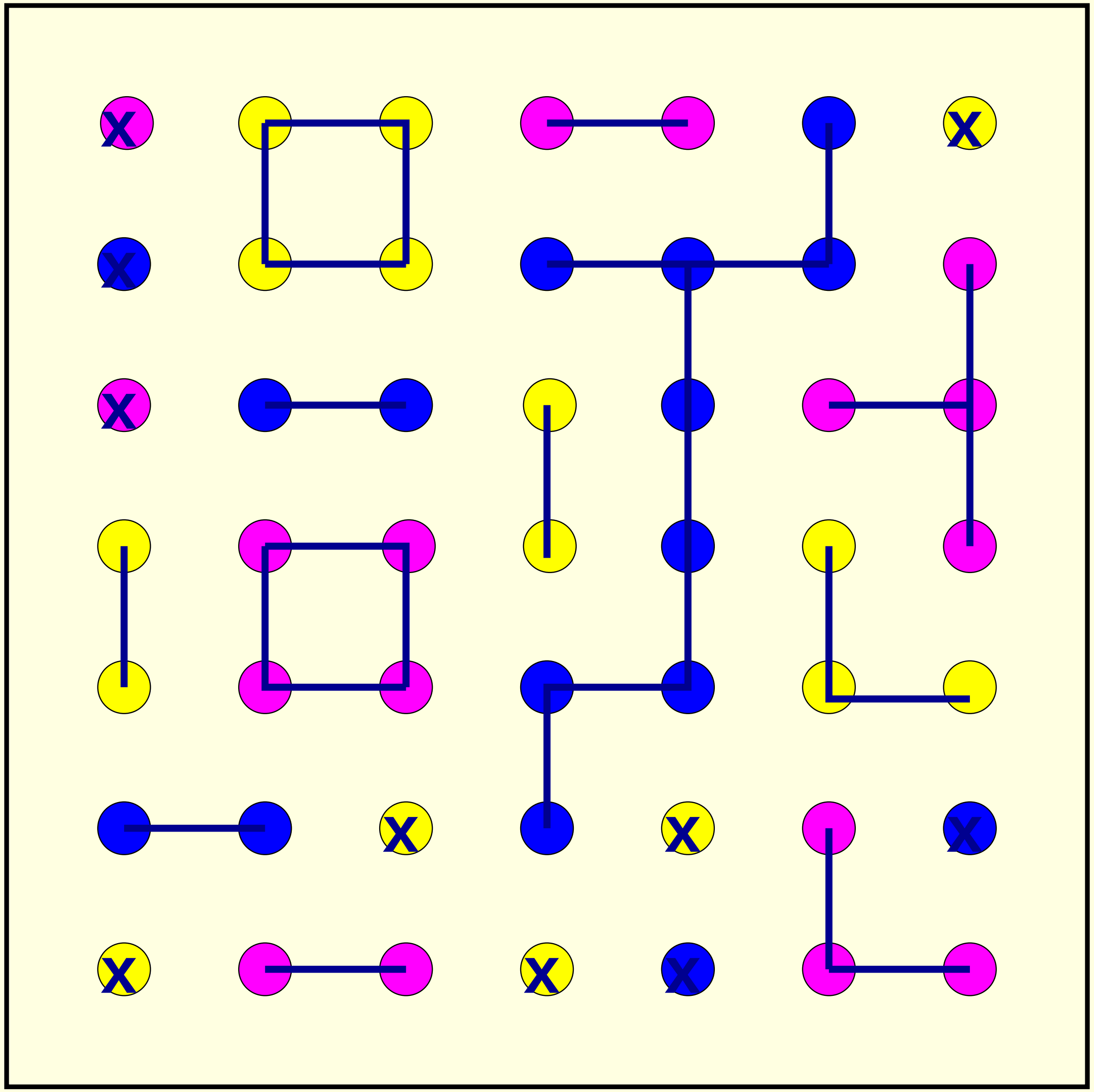
Random cluster representation

$$Z = \sum_{G\subseteq E} q^{c(G)} \prod_{ij\in G} \nu_{ij} \qquad \nu_{ij} = e^{\beta J_{ij}} - 1$$

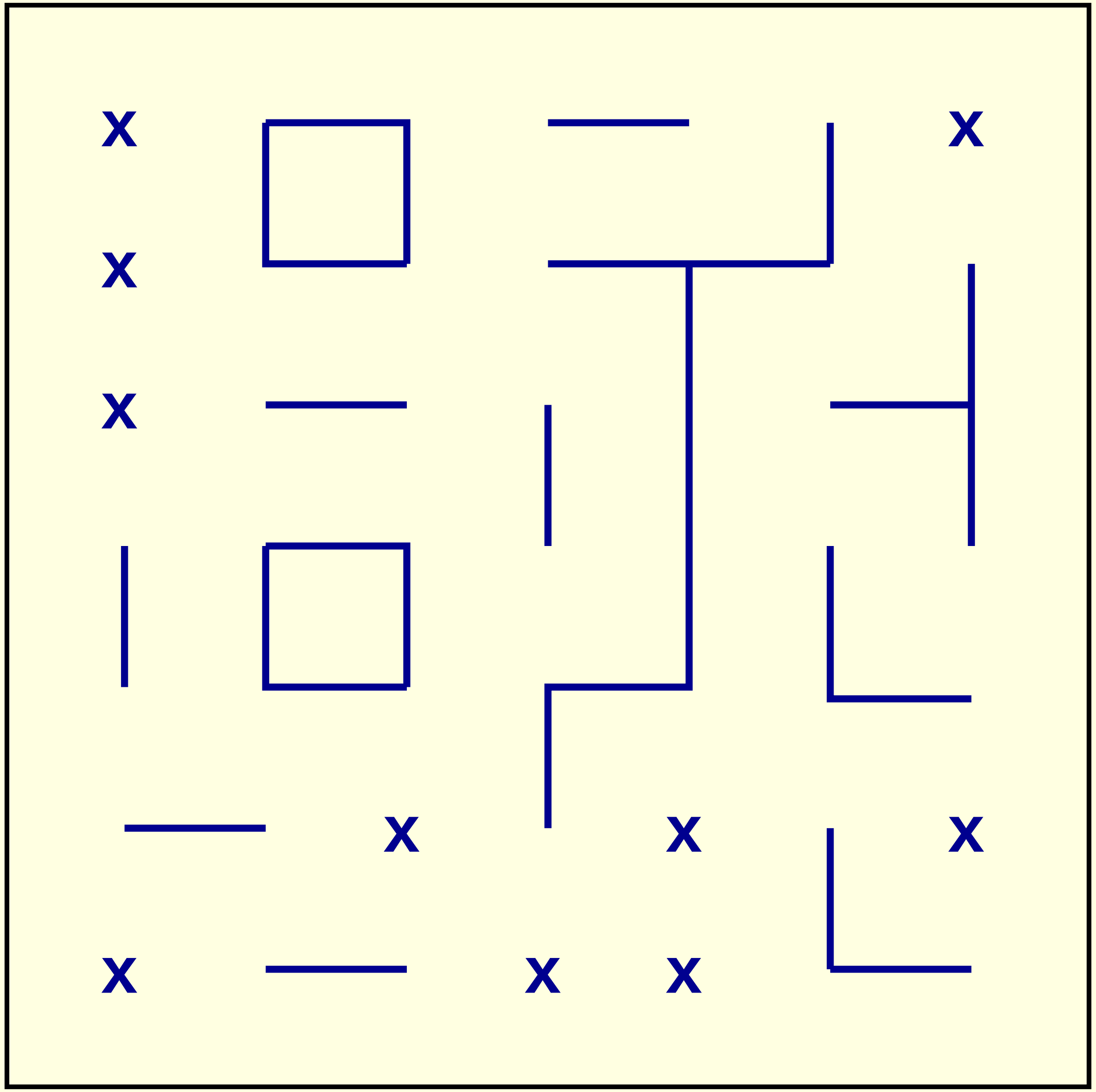
$q=3$



$$c(G) = 10 + 12 = 22$$



$c(G)$ $\prod_e (e^{\beta J_e} - 1)$



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Continuous PT

HARRIS criterion

$\alpha_P > 0$ disorder is **relevant**

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$\alpha_P < 0$ disorder is **irrelevant**	will SOFTEN the transition

Relevant disorder

- Conventional Random FP (different values of critical exponents)
- Infinite Randomness FP (IRFP) (disorder grows without limits)

$q \longrightarrow \infty$ limit

$$T \to T' = T \ln q \qquad\qquad f(T) \to \frac{f(T')}{\ln q}$$

$$Z = \sum_{G \subseteq E} q^{c(G)} \prod_{ij \in G} \left[q^{\beta J_{ij}} - 1 \right]$$

$$\Downarrow_{q \to \infty}$$

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$Z = n_0 q^{\phi^*}(1 + \ldots)$ where $\phi^* = \max_G \phi(G)$ and $\phi^* = -\beta N f$

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  - → ξ, correlation lenght, is the average size of the clusters

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  Angles d'Auriac *et al.*JPA**35**, 6973 (2002)

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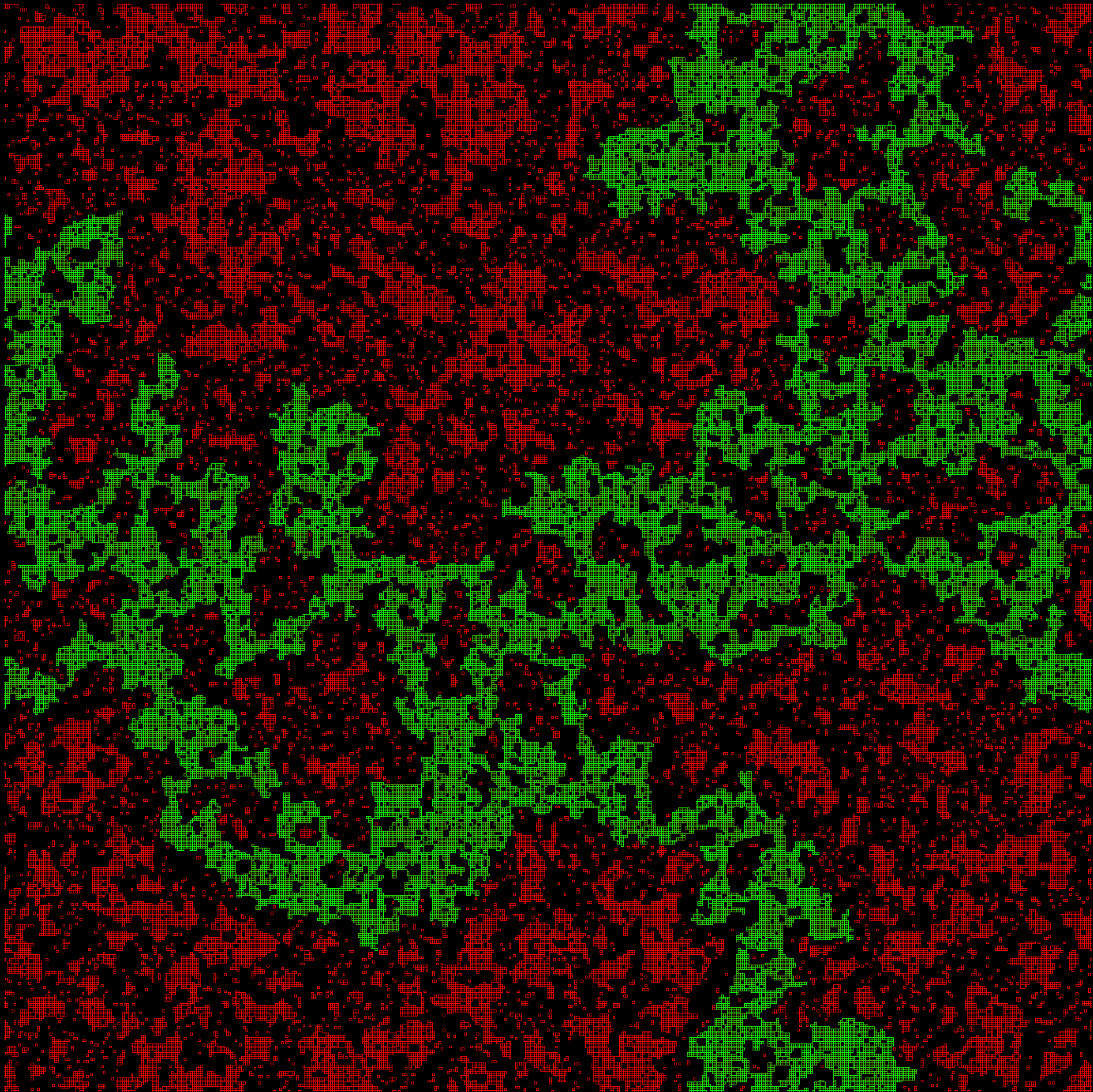
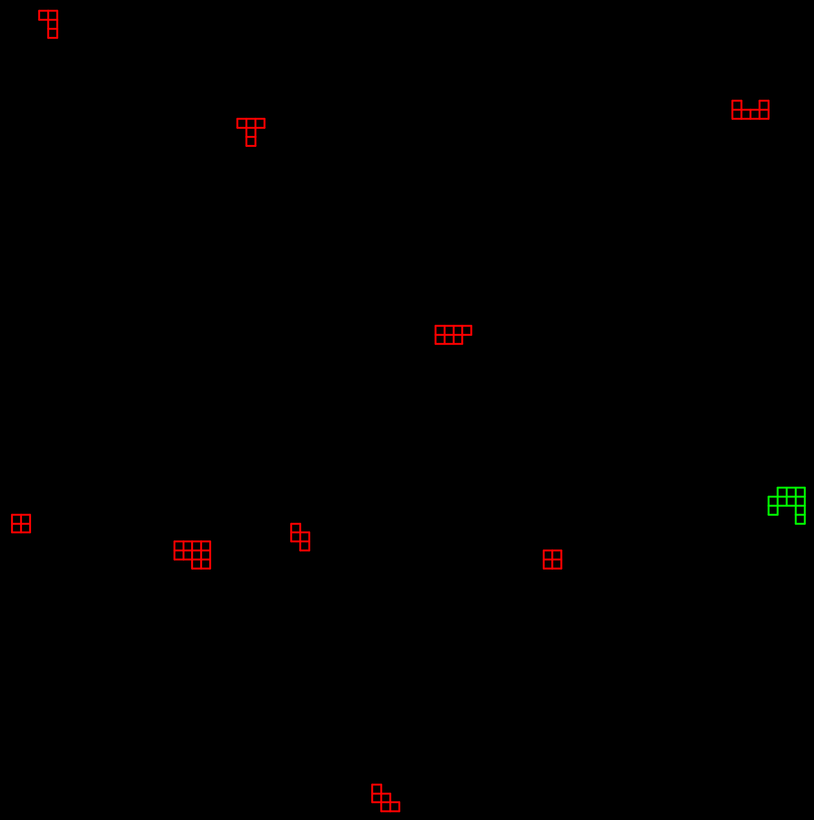
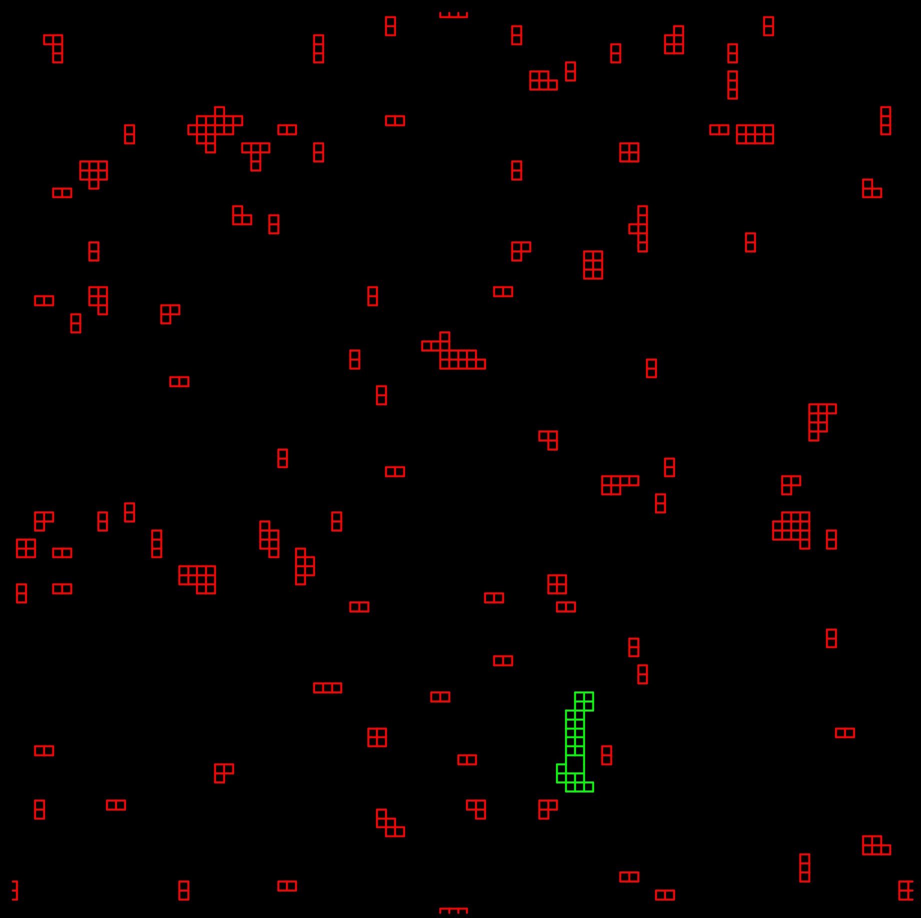
$$d = 2 \qquad L = 512 \quad \Rightarrow \quad 2^{524288} \sim 2.6 \cdot 10^{157826}$$



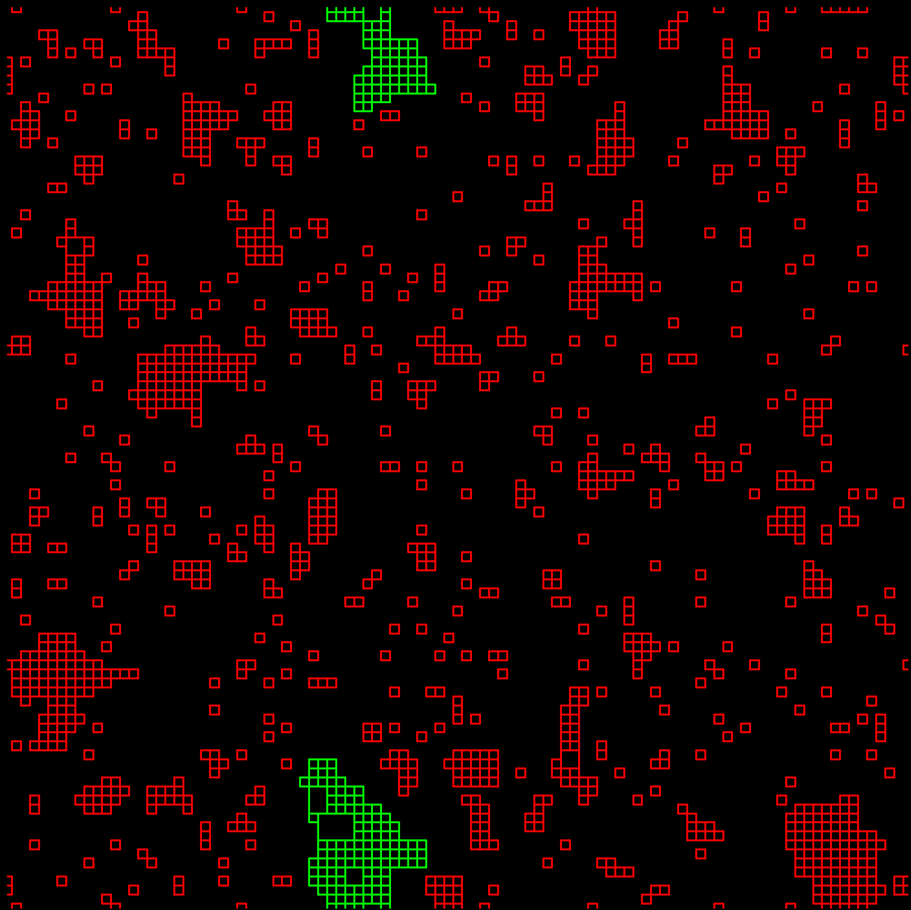
$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 1.200$$



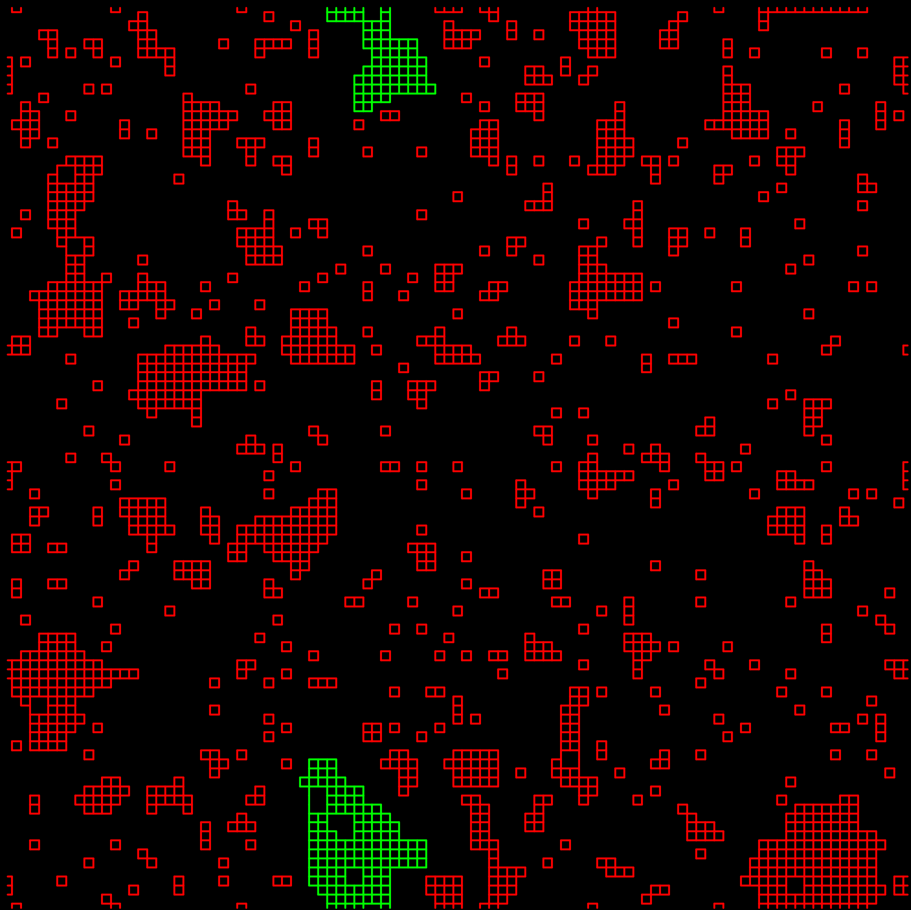
$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 1.166$$



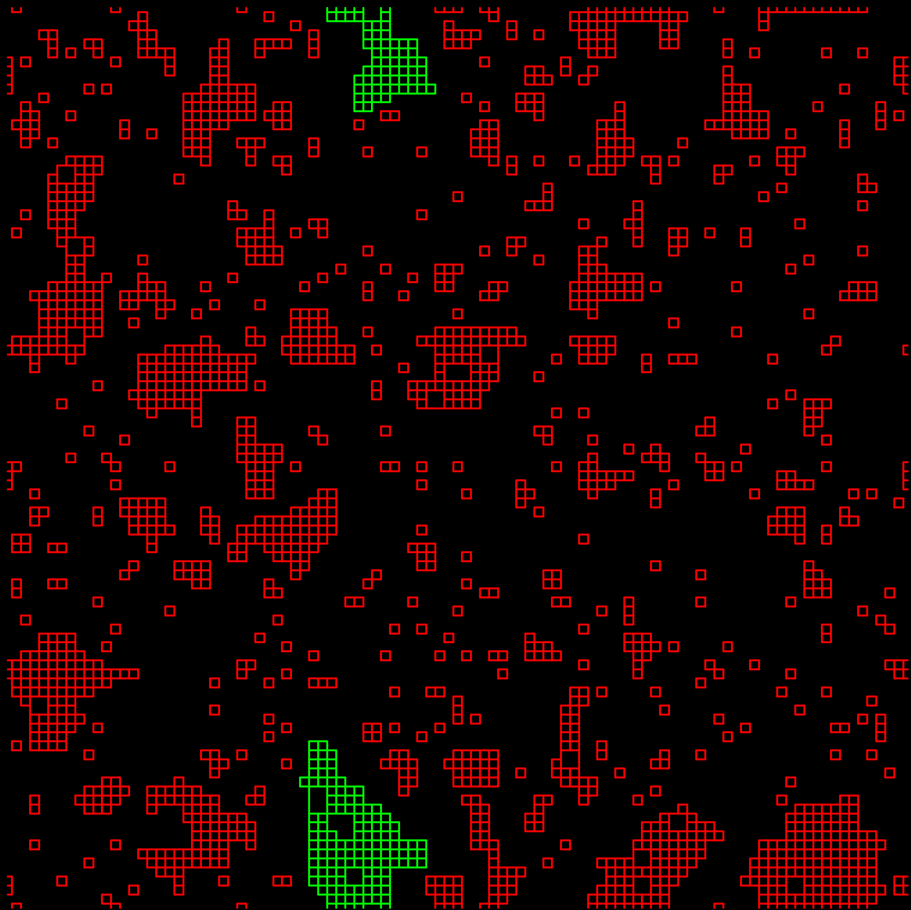
$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 1.066$$



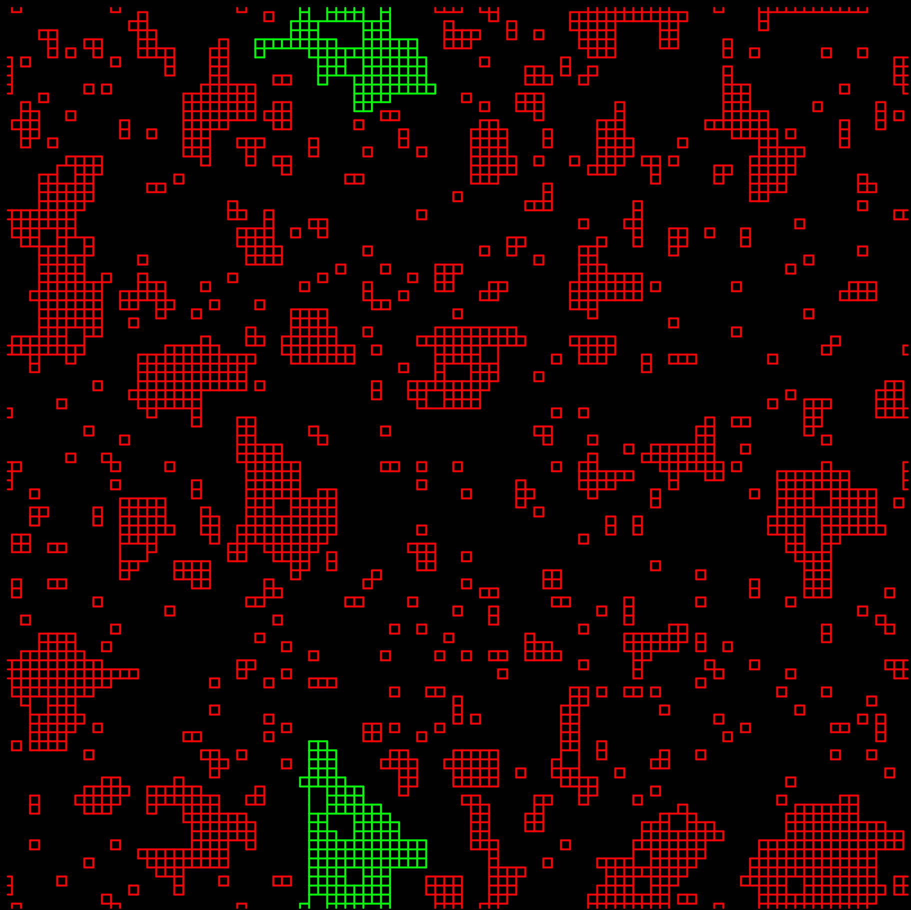
$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 1.050$$



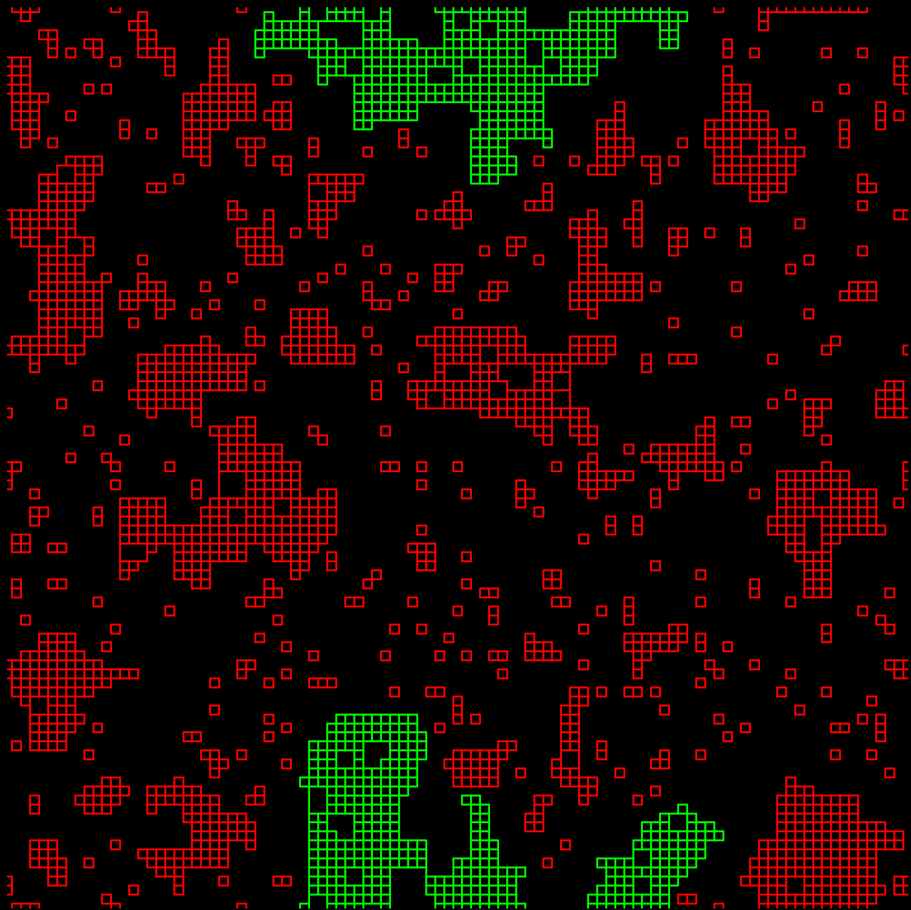
$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 1.042$$



$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 1.033$$



$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 1.025$$



$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 1.016$$

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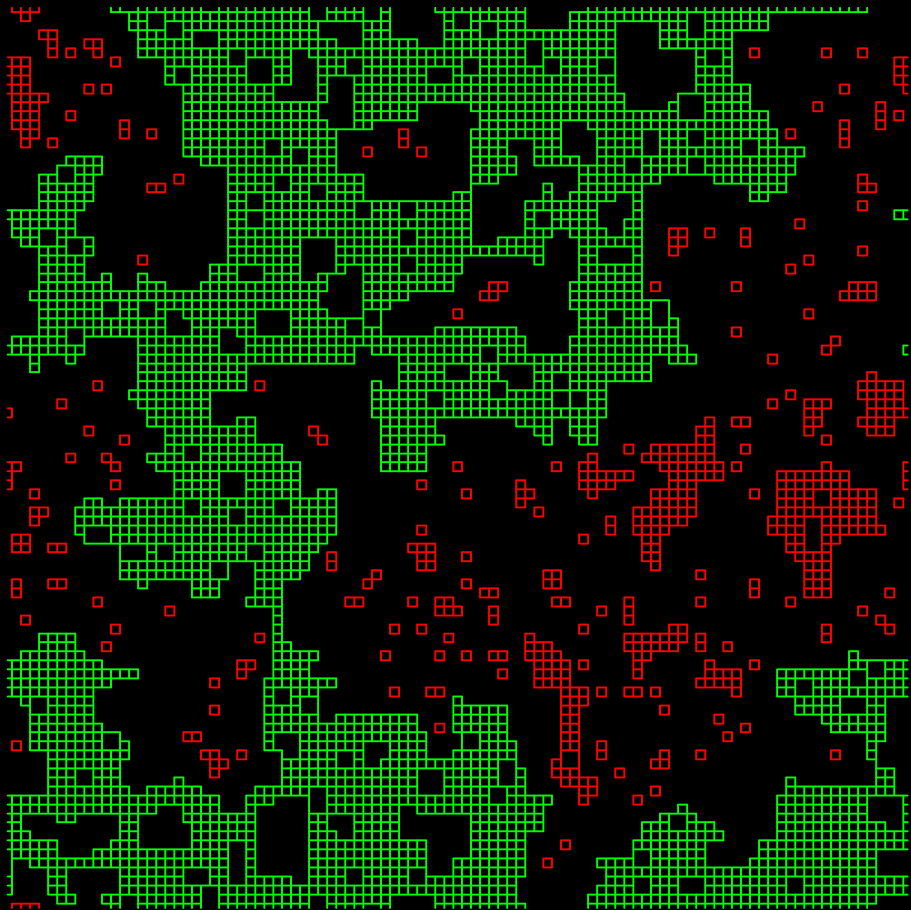

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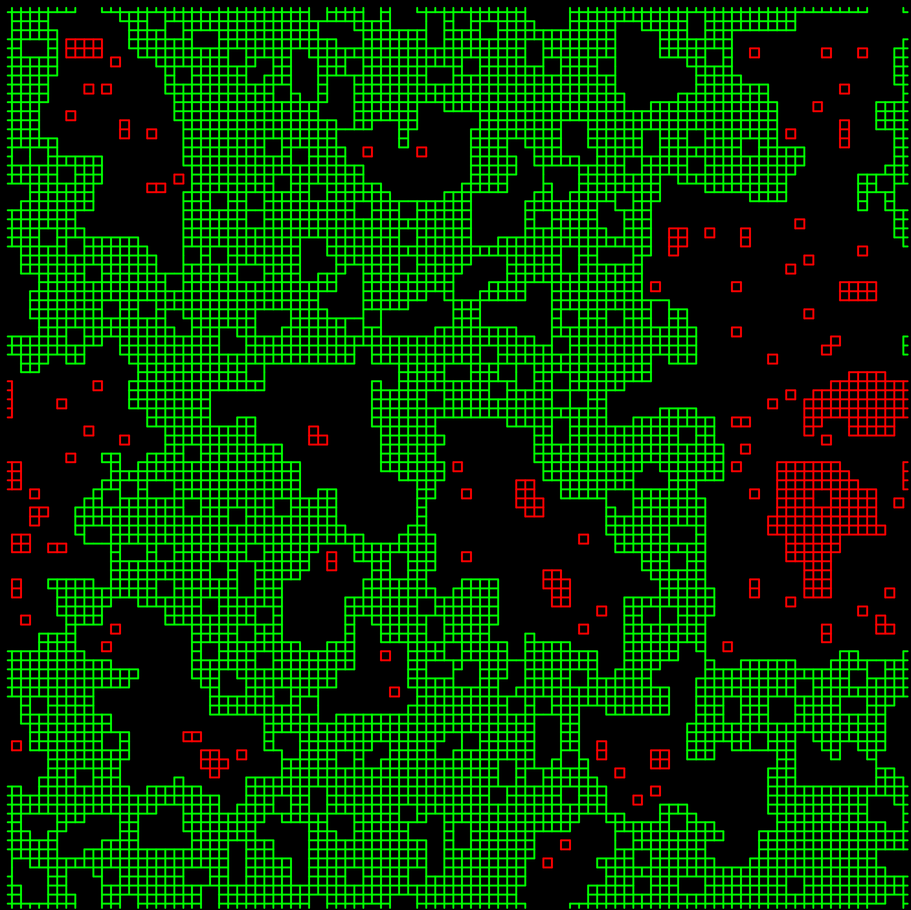
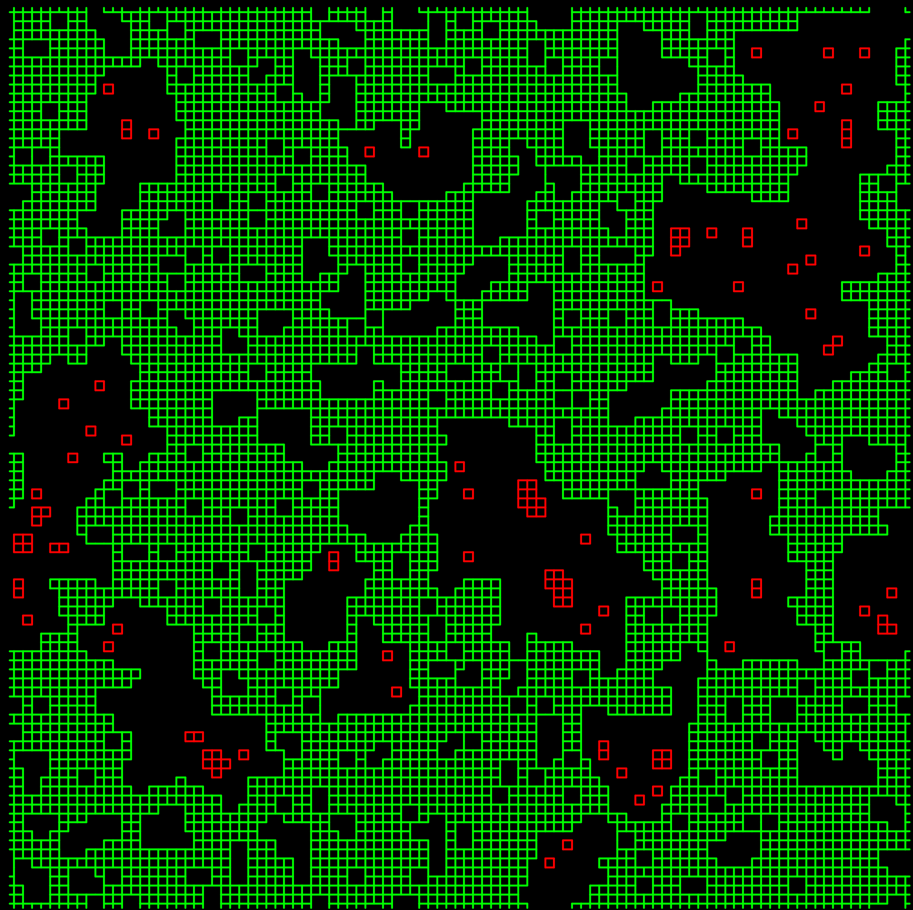
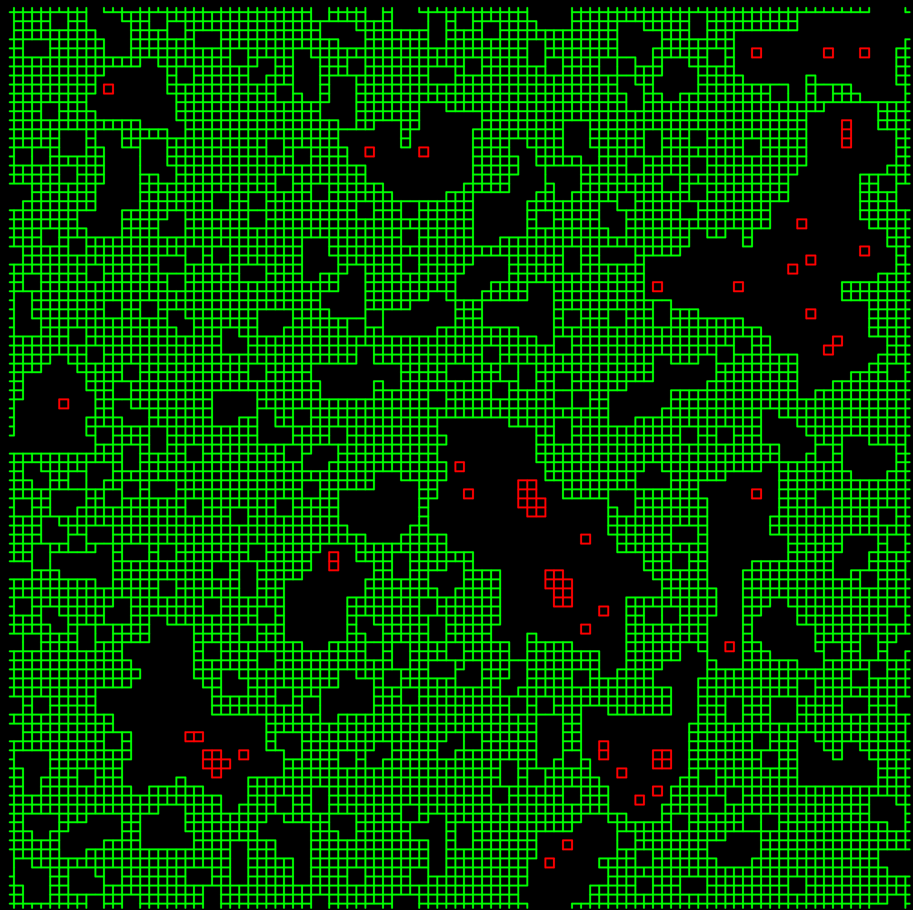


$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 0.992$$



$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 0.983$$



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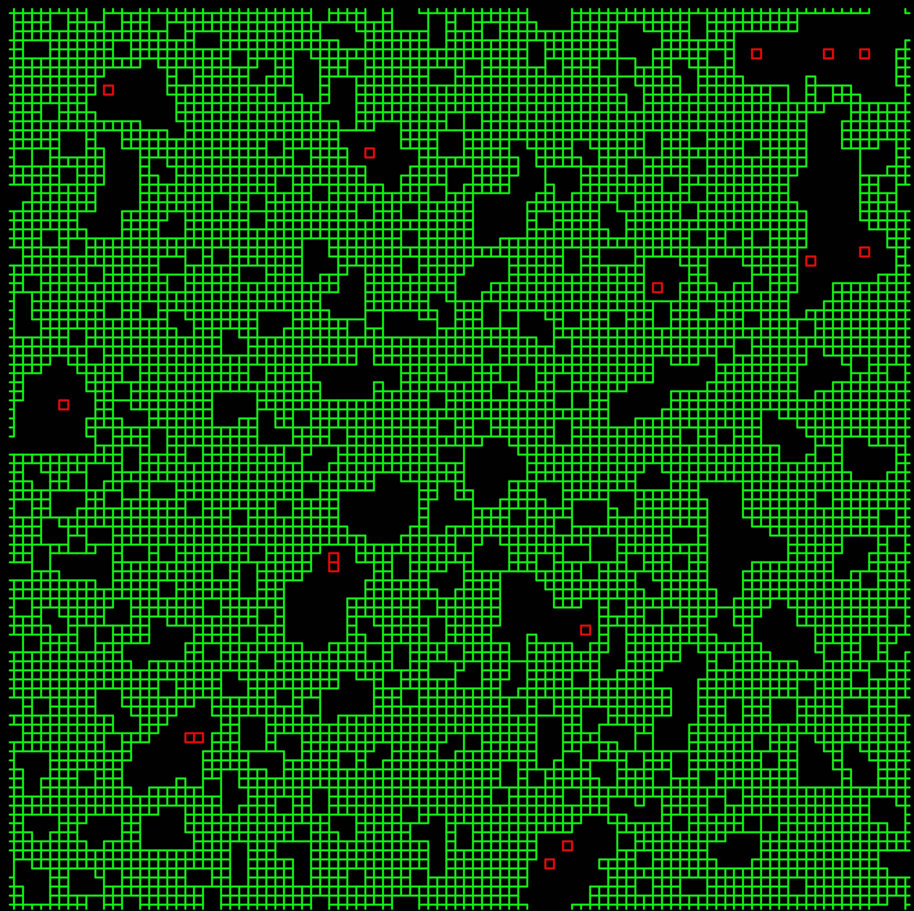
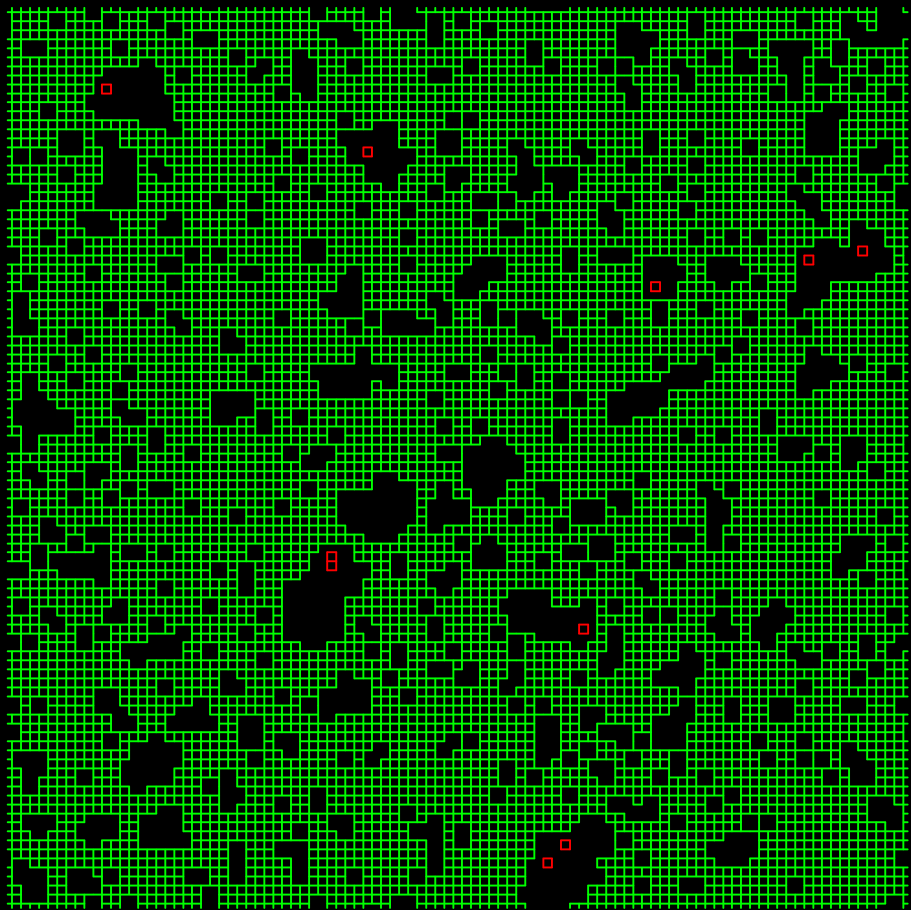


$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 0.9416$$



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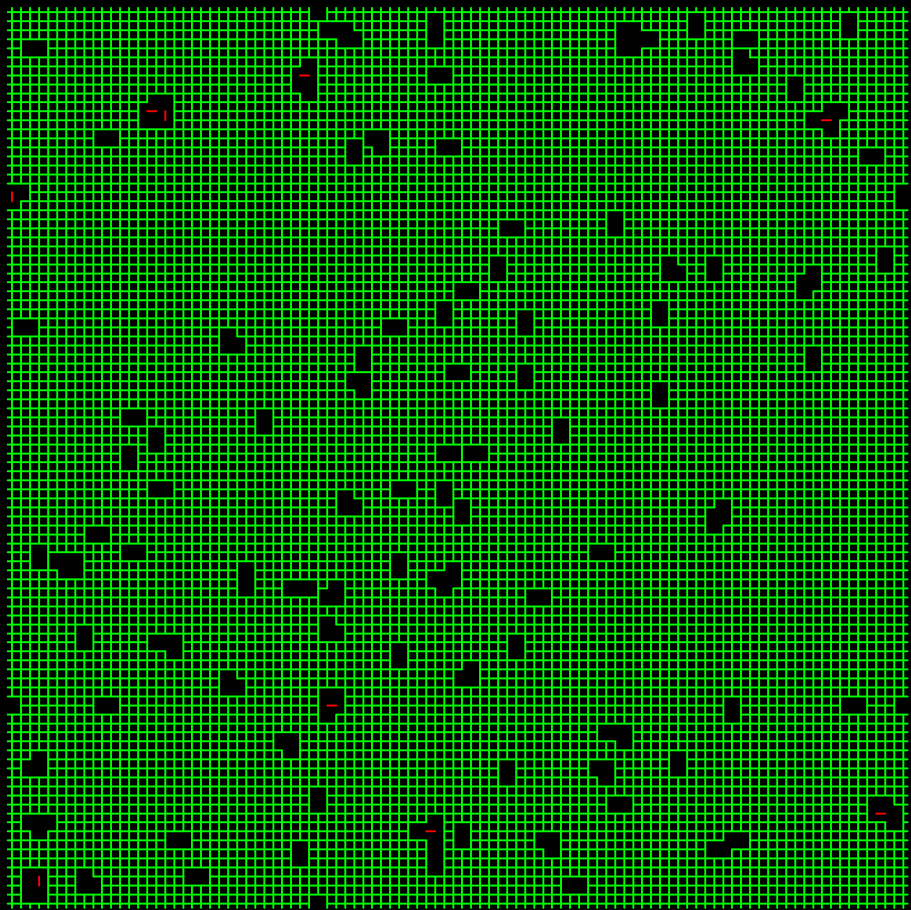
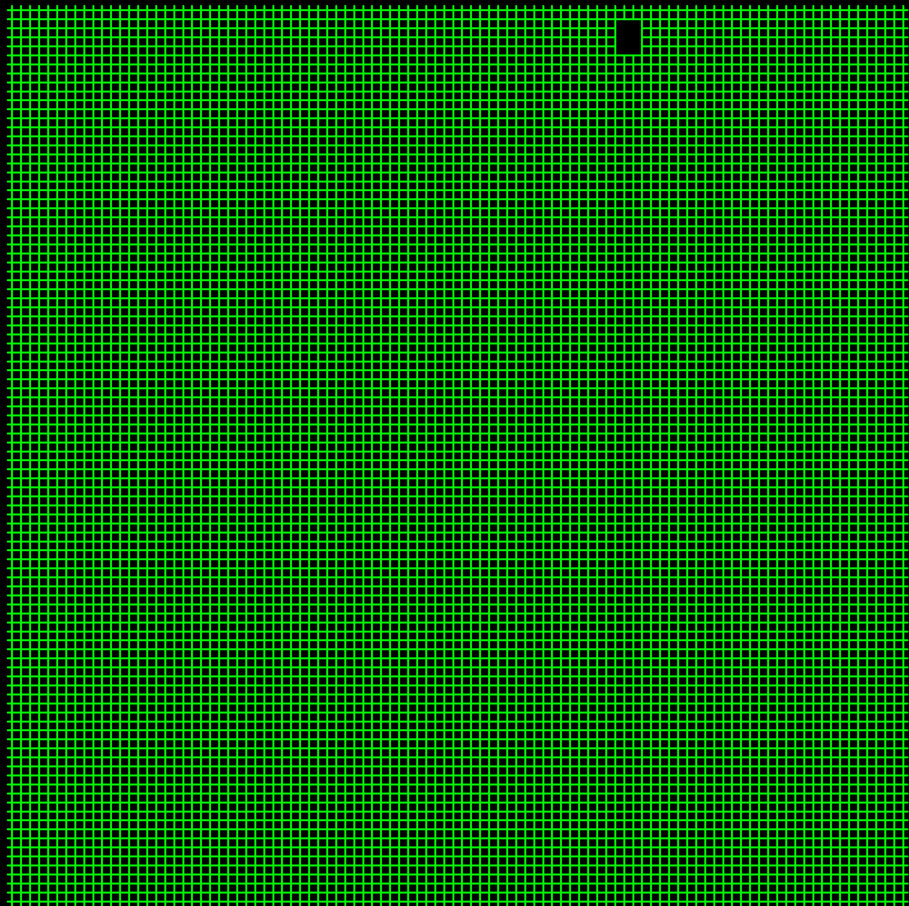


$$P(J) = \frac{1}{2}\delta\left(J - \frac{1}{6}\right) + \frac{1}{2}\delta\left(J - \frac{5}{6}\right) \qquad T = 0.500$$



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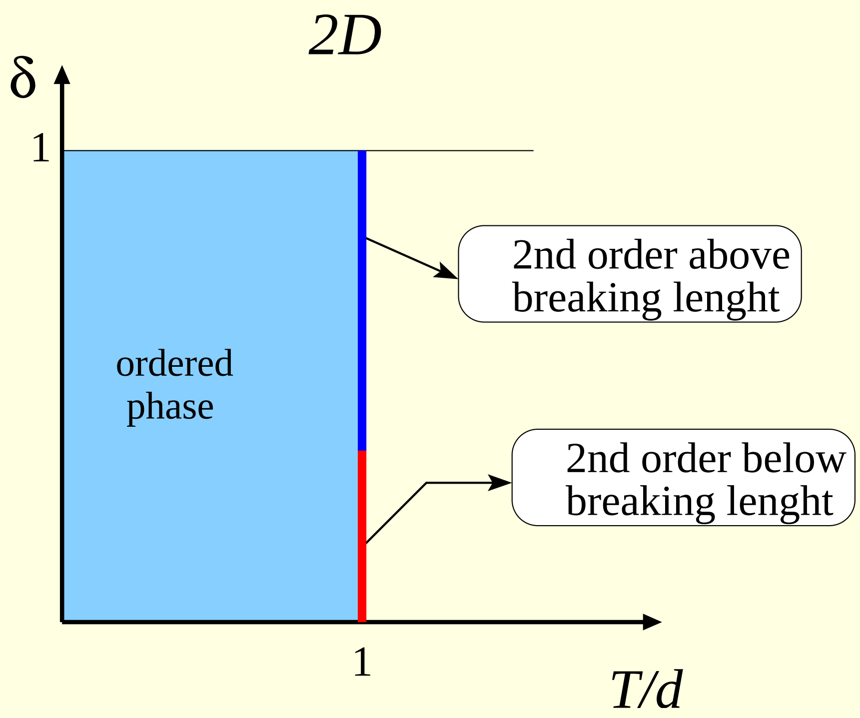

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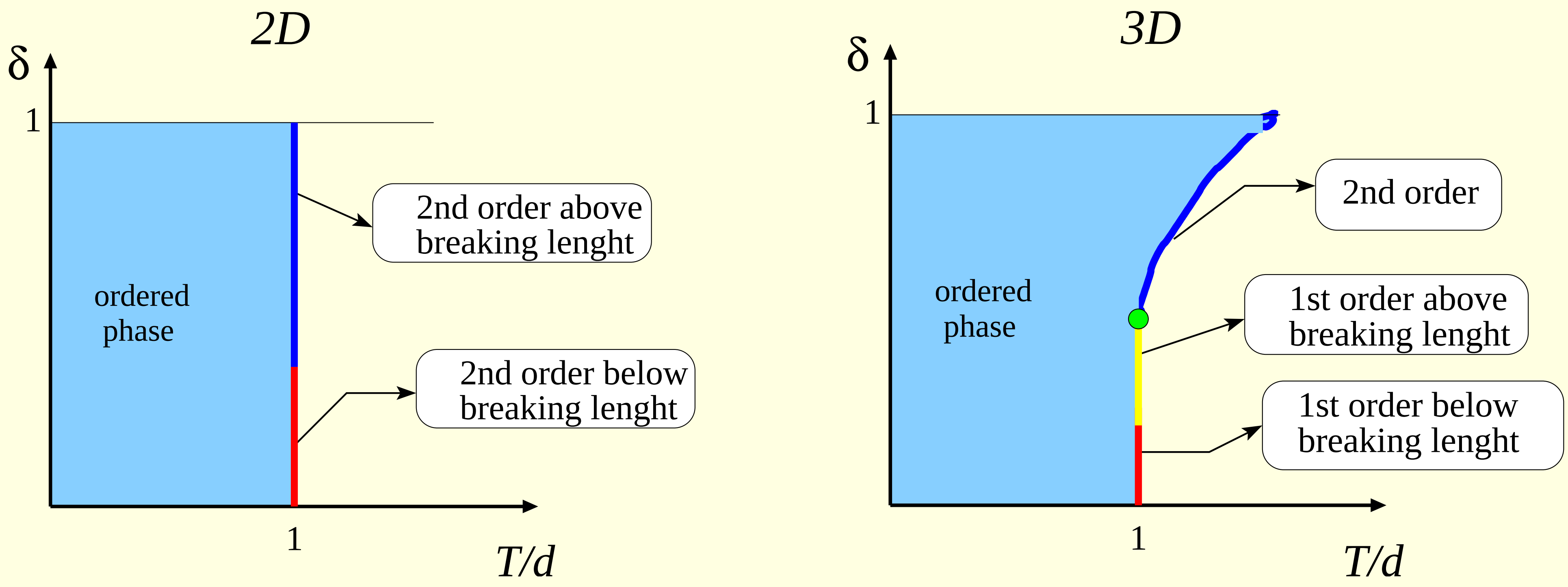

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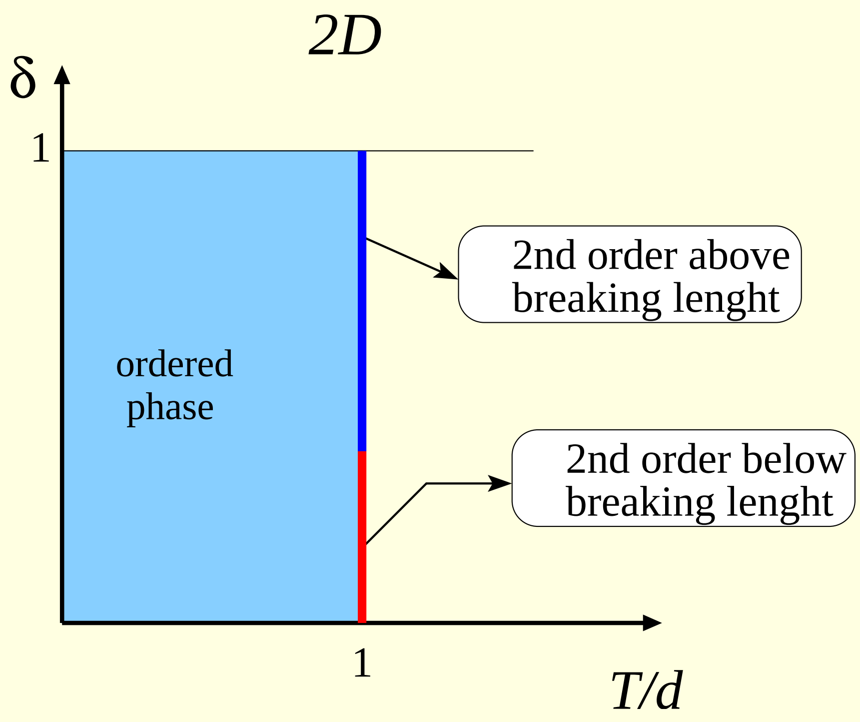

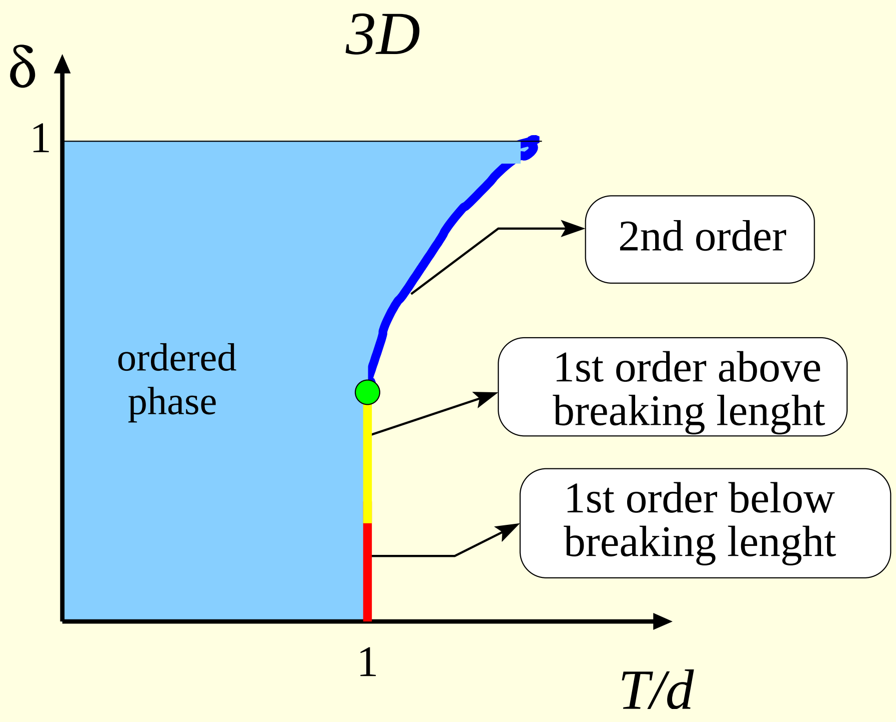

In a finite size system weak disorder fluctuation could not be sufficient to break phase coexistence

Through extreme value statistics one can estimates the **breaking length scale** $L \sim \exp[(1/\delta)^2]$

[the finite length scale L at which breaking of phase coexistence take place]

strenght of disorder $\delta = \Delta/J$

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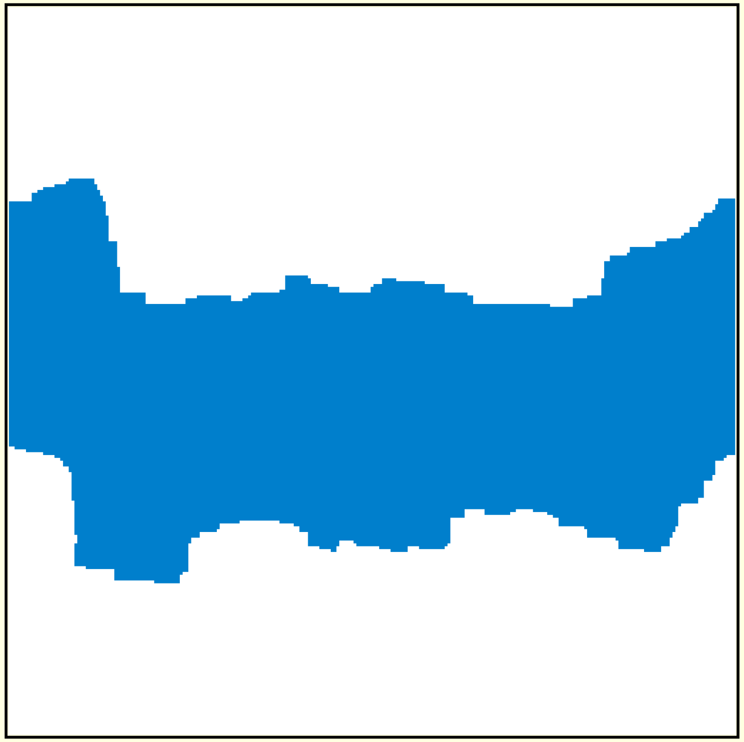

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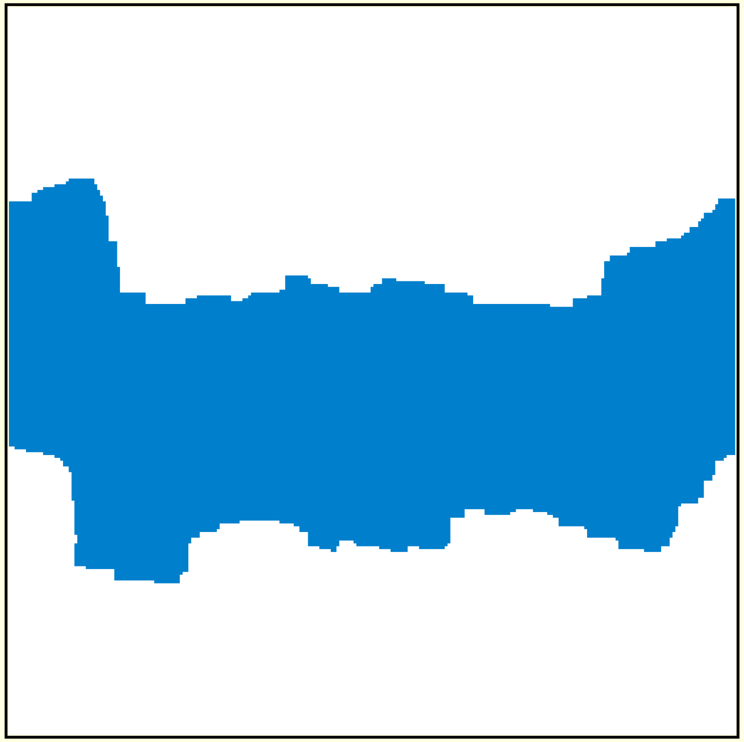

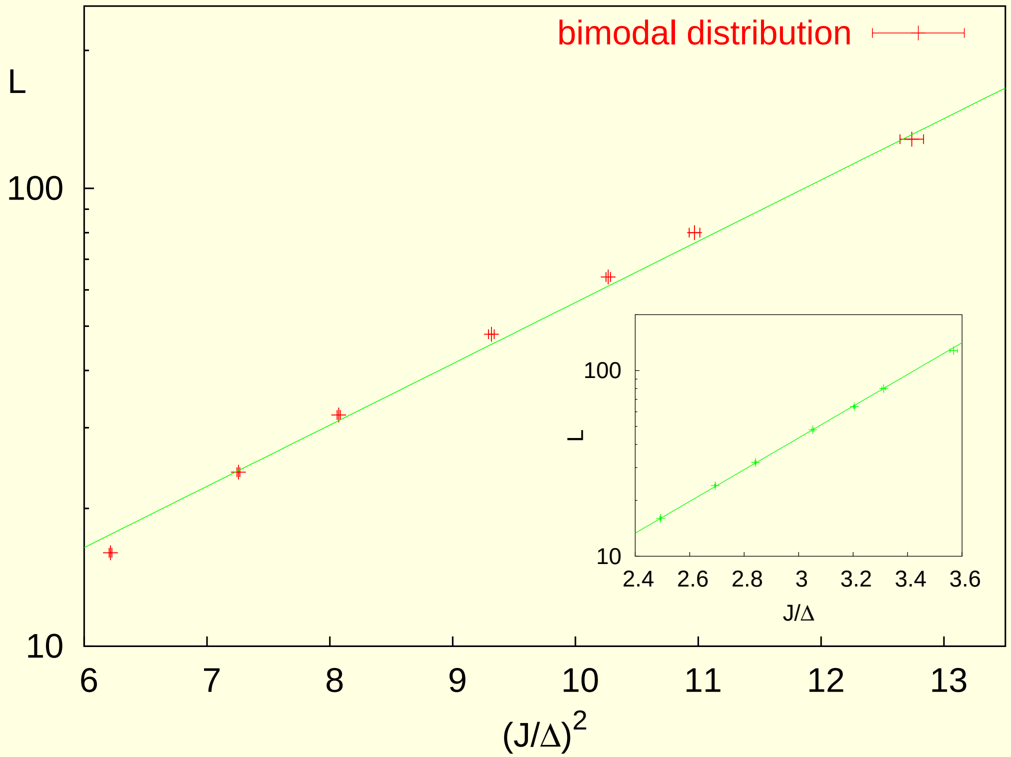

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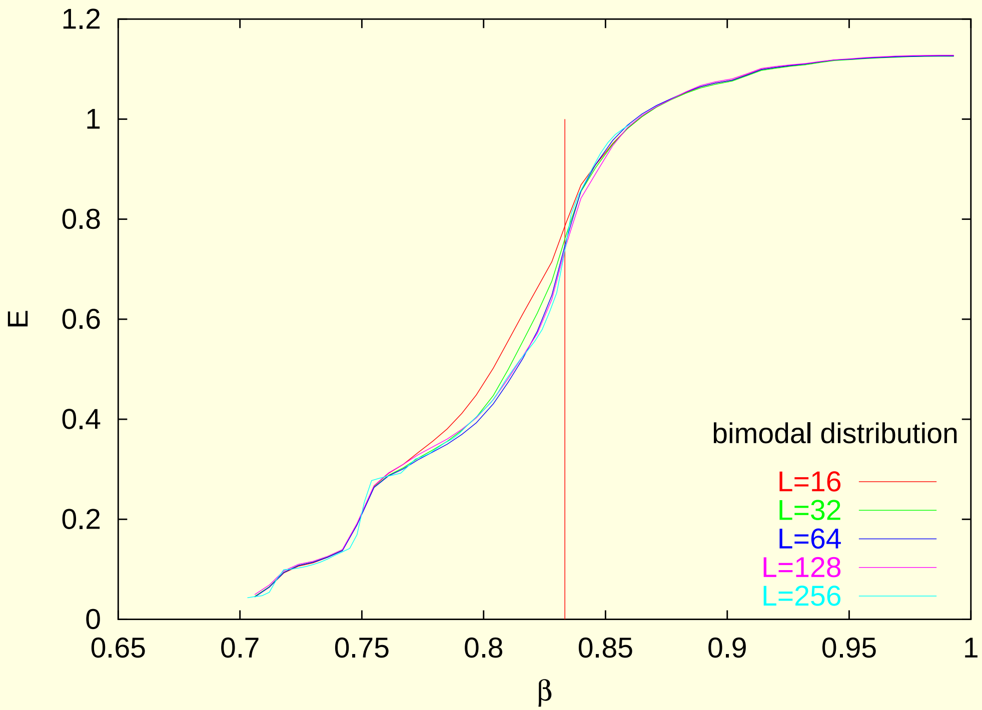
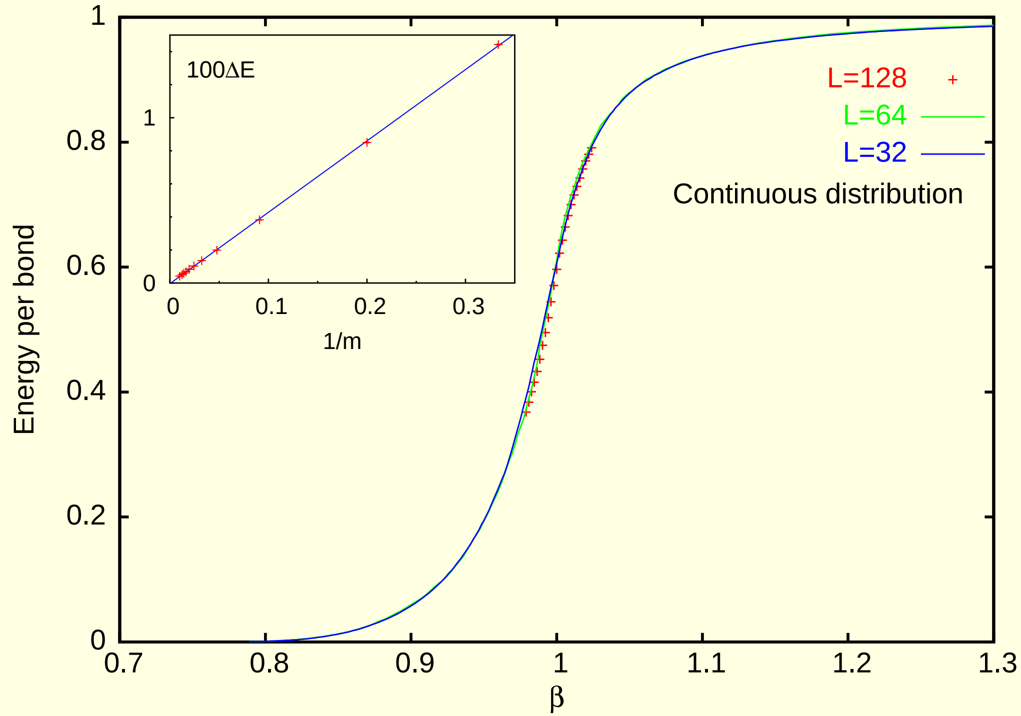
Free Energy: $F = c(G^*)T - \sum_{ij \in G^*} J_{ij}$ Internal Energy: $E = -\sum_{ij \in G^*} J_{ij}$

- For a given sample E is a piecewise costant function of temperature $\Rightarrow$ it shows discontinuities
- The average over disorder generally smears out discontinuities
- The behavior of averaged quantities is different for the discrete and the continuous distributions



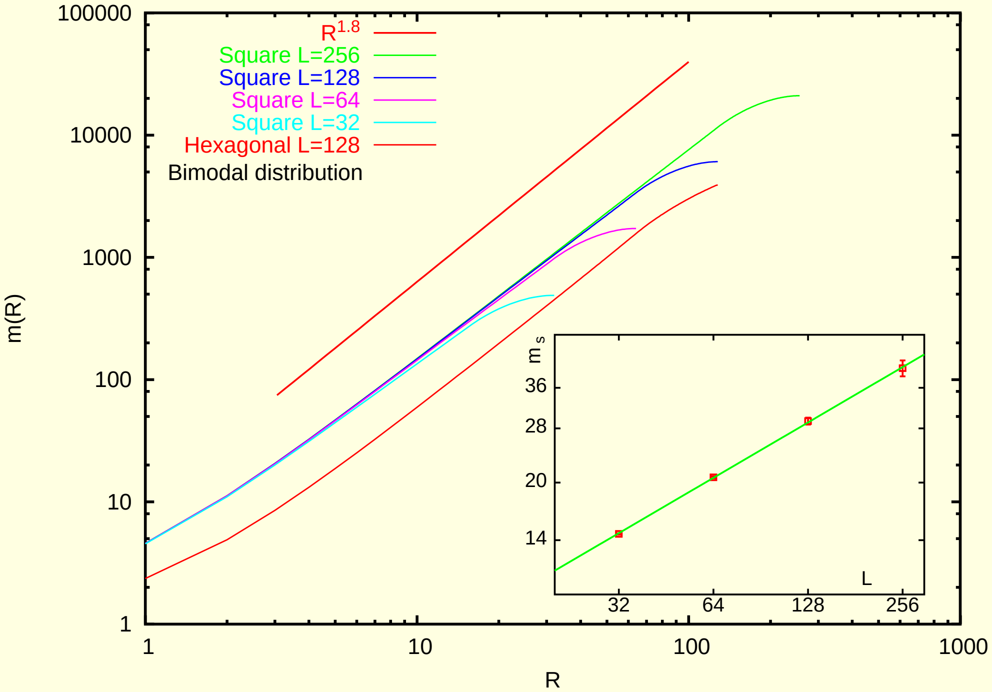


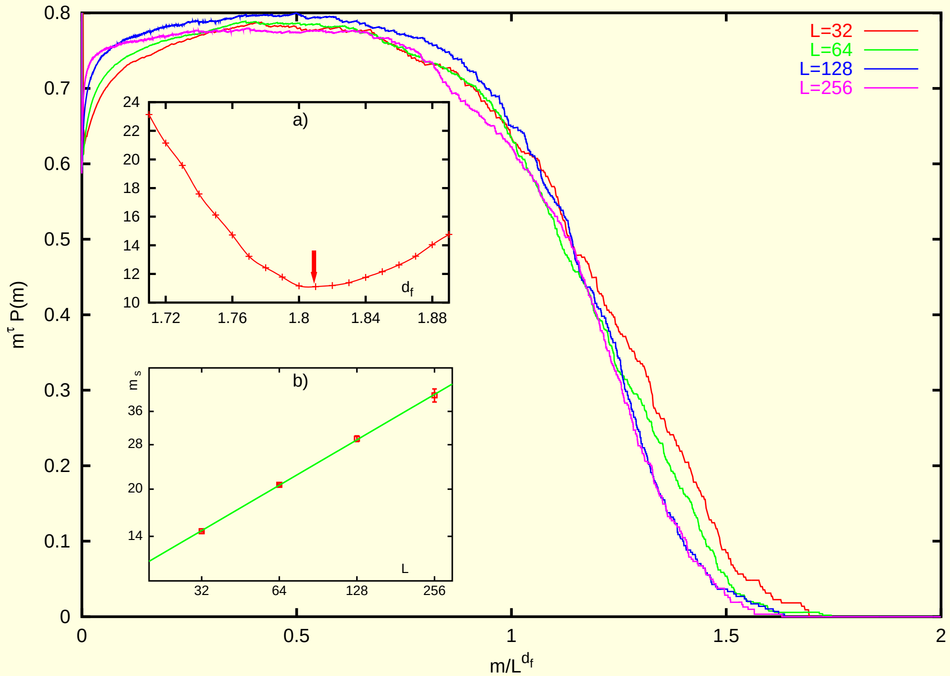
At the critical point the largest cluster of G^* is a fractal and its mass $M \sim L^{d_f}$

$$d_f = d - \frac{\beta}{\nu} \qquad d_f = \frac{5+\sqrt{5}}{4}$$

According to scaling theory, cumulative distribution of the mass of the cluster

$$R(M,L) = M^{-\tau}\tilde{R}(M/L^{d_f})$$





Conclusions

➠ Results in 2D

→ $\alpha = 0, \beta = \dfrac{3-\sqrt{5}}{4}, \nu = 1$ as for the RTIM $\Rightarrow$ IRFP !

→ We can argue that the RTIM is the Hamiltonian version of the 2D RBPM in the large-q limit

Ref.: Mercaldo, Anglès d'Auriac, Iglói, PRE **69**, 0461xx (2004);

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→ is the transition line $T_c/d = 1$ for $\delta \ll 1$?

→ is $\delta = 1/2$ the tricritical point ?

→ does the critical line depend on the disorder distribution ?

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Related Problem

- Critical Properties of Quantum Potts model

  Ref.: Mercaldo, De Cesare, work in progress