# Multi-dimensional Piecewise Deterministic Markov Processes: a first order numerical treatment.

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We consider a numerical treatment of the following Liouville - Master Equation (LME):

$$\partial_t F_s(\overrightarrow{x},t) + \sum_{k=1}^{d} A_s^{(k)}(x_k) \cdot \partial_{x_k} F_s(\overrightarrow{x},t) = \sum_{j=1}^{S} Q_{sj} F_j(\overrightarrow{x},t), \qquad (1)$$

for the unknown distribution functions $F_s(\overrightarrow{x},t)$, $\overrightarrow{x} = x_1, \dots x_d$. Eq. (1) is related to a $d$-dimensional piecewise deterministic Markov process [1], described by the system of ODE's:

$$\frac{dX_k}{dt} = A_s^{(k)}(X_k) \qquad k = 1, \dots, d, \qquad (2)$$

where $A_s^{(k)}(X_k)$ is chosen randomly from a set of $s = 1, \dots, S$ known functions with $d$-components and one independent variable, and it is subject to a Markov process of stochastic transition matrix $q_{ij}$, $i,j = 1, \dots, S$, and Poisson statistics of transition rates $\mu_s$. The importance of LME is that it provides an alternative way to a direct Monte Carlo simulation of Eq. (2) when extracting the statistical properties of the process. Eq. (1) is solved for Cauchy conditions $F_s(\overrightarrow{x},0) = F_s^{(0)}(\overrightarrow{x})$, and boundary conditions: $\lim_{\{x_1,\dots,x_d\}\to\infty} \sum_s F_s(\overrightarrow{x},t) = 1$, $\lim_{\{x_1,\dots,x_d\}\to-\infty} F_s(\overrightarrow{x},t) = 0$, $\lim_{x_k\to\infty} \sum_s F_s(x_1,\dots,x_k,\dots,x_d,t) \le 1$. The one-dimensional case has been studied in [2], for which convergence and monotonicy has been prooved and tested for the upwind method, under the Courant-Friedrichs-Lewy (CFL) condition. An extended CFL condition can guarantee that the upwind produces a convergent solution for the $d$-dimensional case (1). Some numerical tests are performed for $d = 2$, showing the time dependent density probability distribution function $p(x_1,x_2,t) = \partial_{x_1 x_2} \sum_s F_s(x_1,x_2,t)$ for processes having a statistical equilibrium.

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## References

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