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A NUMERICAL TREATMENT OF THE LIOUVILLE-MASTER EQUATION FOR PIECEWISE DETERMINISTIC PROCESSES WITH MEMORY: CONVERGENCE AND MONOTONICITY

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We study the numerical treatment of the Liouville-Master equation for the probability distribution functions $F_s(x,y,t)$ of a continuous *piecewise-deterministic process* (PDP) [1; 2]; such equation read as a system of hyperbolic PDEs [3]:

$$\partial_t F_s(x,y,t) + A_s(x)\,\partial_x F_s(x,y,t) + \partial_y F_s(x,y,t) = -\lambda_s(y)\,F_s(x,y,t) \tag{1}$$

with Cauchy initial conditions: $F_s(x,y,t_0) = F_{0,s}(x)\delta(y)$, and non-local boundary conditions:

$$F_s(x,0,t) = \sum_{j=1}^{S} q_{sj} \int_0^{t-t_0} F_j(x,y,t)\,\lambda_j(y)\,dy \tag{2}$$

for the $s = \{1,\ldots,S\}$ unknowns $F_s : \mathcal{D} \to \mathbb{R}$, with $(x,y,t) \in \mathcal{D} := (\Omega \times [0,T-t_0] \times [t_0,T]) \subset \mathbb{R}^3$, where $\Omega = [\Omega_a,\Omega_b] \subset \mathbb{R}$. Here $A_s(x)$ are known deterministic functions, $\lambda_s(y)$ are a set of *hazard functions* and q_{sj} are the elements of a stochastic matrix having the following fundamental properties: $0 \le q_{sj} \le 1$ and $\sum_s q_{sj} = 1$.

The numerical approximation ${}^sF^n_{k,j}$ to $F_s(x_k,y_j,t_n)$ is calculated by a combination of the *upwind* method, for the Eq. (1), and a first order direct quadrature, for the Eq. (2). We show that under a Courant-Friedrichs-Lewy condition: (i) the global error $\hat{E}^n_k$ associated to the distribution function at time t_n and position x_k, regardless sates s and memory y, i.e. $\int_0^{t_n} \sum_s F_s(x_k,y_j,t_n)dy$, is first order convergent [3]: $\|\hat{E}^n\|_\infty \propto \mathcal{O}(\Delta x) K_2 t_n^2 \exp(K_1 t_n)$; (ii) ${}^sF^n_{k,j}$ is non decreasing with respect to k and positive w.r. to j, when the Cauchy initial condition is non decreasing [4].

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REFERENCES

[1] M.H.A. Davis. *Markov Models and Optimizations.* Chapman & Hall/CRC, London 1993.

[2] G.N. Milstein and Yu.M. Repin. Action of Markov process on a system of differential equations. *Differential Equations*, **5** (1972) pp. 1010–1019.

[3] M. Annunziato. A finite difference method for piecewise deterministic processes with memory. *Mathematical Modelling and Analysis*, **12** (2) 2007, 157-178.

[4] M. Annunziato. On a finite difference method for piecewise deterministic Markov processes and its parallel algorithm implementation. *Preprint*, **12** 2006. Dipartimento di Matematica e Informatica-Università degli Studi di Salerno.